MA 141-012 

Wednesday, November 28

TEST #4 (first hour)

5.2: (second hour)

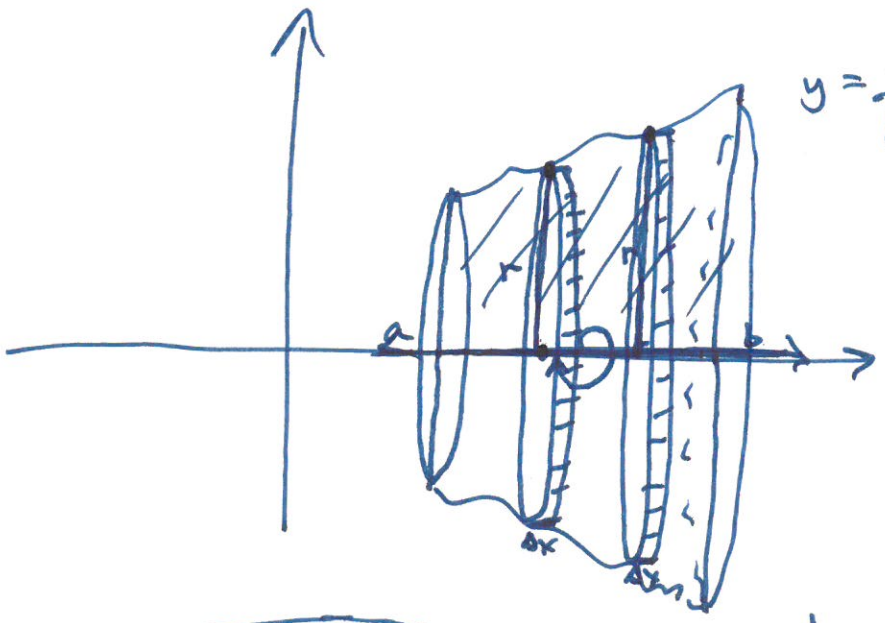
## VOLUMES OF SOLIDS OF REVOLUTION

## (VOLUMES "BY SLICING")

(1) SOLID DISKS

(2) WASHER

(3) CYLINDRICAL SHELL

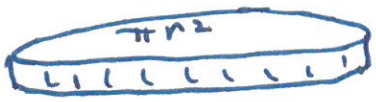



$y = f(x)$

bounded region

revolve about the x-axis

"slice it up" ⊥ to axis of revolution




CYLINDER

$VOL = \pi \cdot r^2 \cdot h$

$$\int_a^b \underbrace{\pi r^2 \cdot h}$$

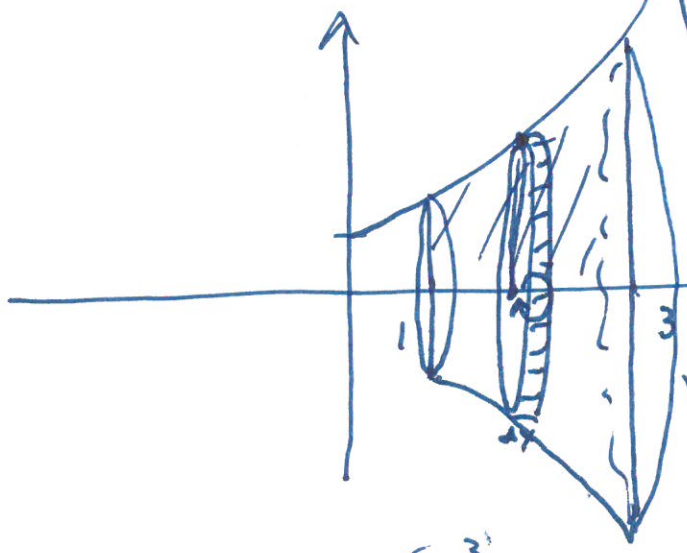
$r = y = f(x)$

$h = \Delta x \to dx$

$$V = \pi \int_a^b (f(x))^2 \cdot dx$$

$f(x) = x^2+1$

volume of the solid of revolution formed when the region bounded by $y = x^2+1$ from $x=1$ to $x=3$ (and the x-axis) is revolved about the x-axis.



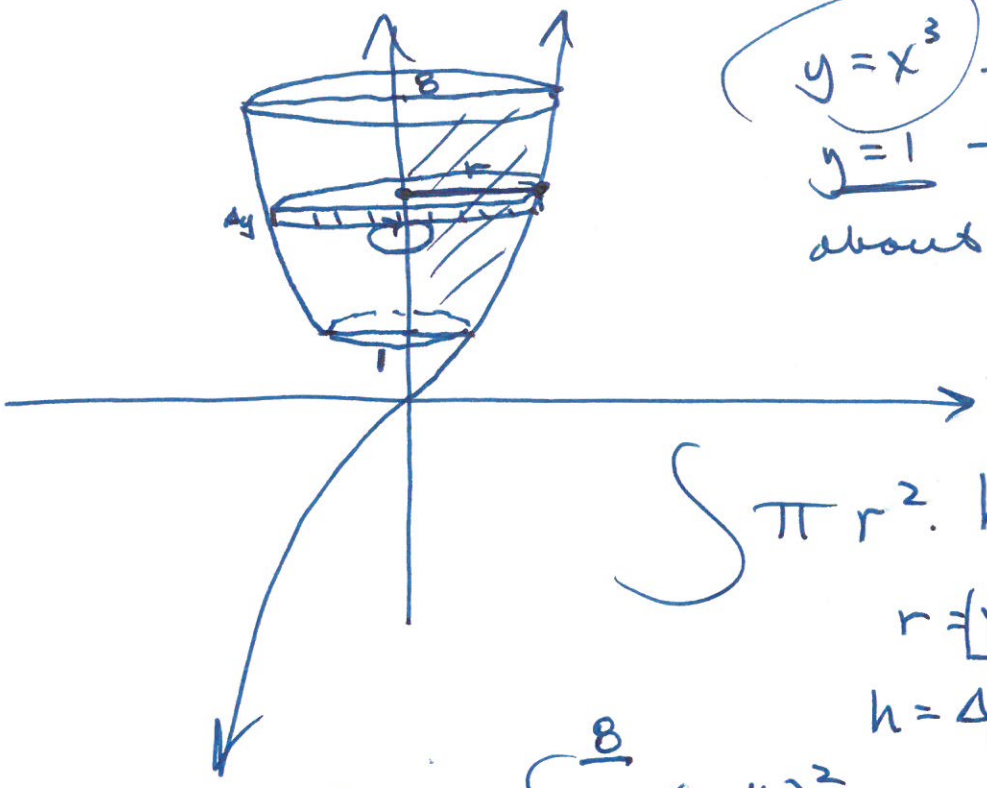

$$V = \int_1^3 \pi (r)^2 \cdot h$$

$$r = y = f(x) = x^2+1$$

$$V = \pi \int_1^3 (x^2+1)^2 \cdot dx$$

$$V = \pi \int_1^3 (x^4 + 2x^2 + 1)\, dx$$

$$= \pi \left[ \frac{x^5}{5} + \frac{2x^3}{3} + x \right]_1^3 = \text{\_\_\_\_\_}$$




(3)

$y = x^3 \rightarrow y^{1/3} = x$

$y = 1$ to $y = 8$

about the y-axis

$$\sum \pi r^2 \cdot h$$

$r = x = y^{1/3}$

$h = \Delta y \rightarrow dy$

$$V = \pi \int_1^8 (y^{1/3})^2 \, dy$$

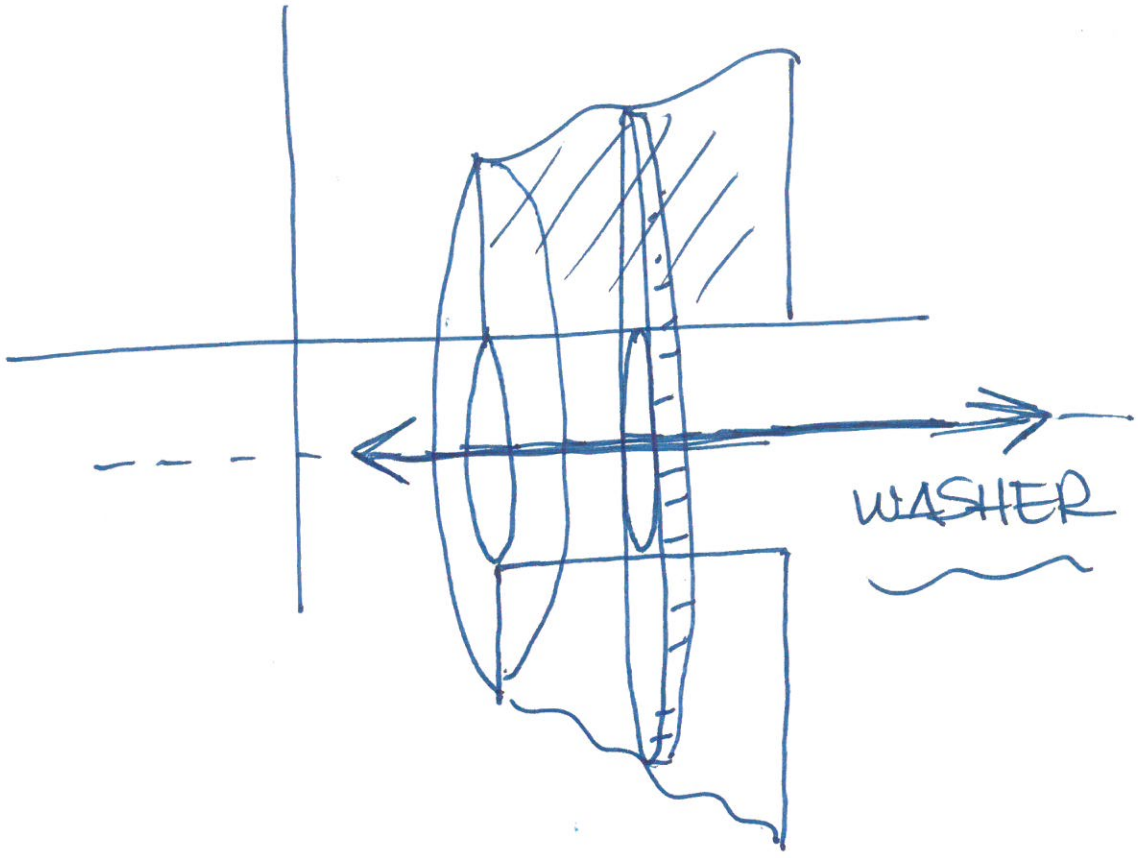
$$V = \pi \int_1^8 y^{2/3} \, dy$$

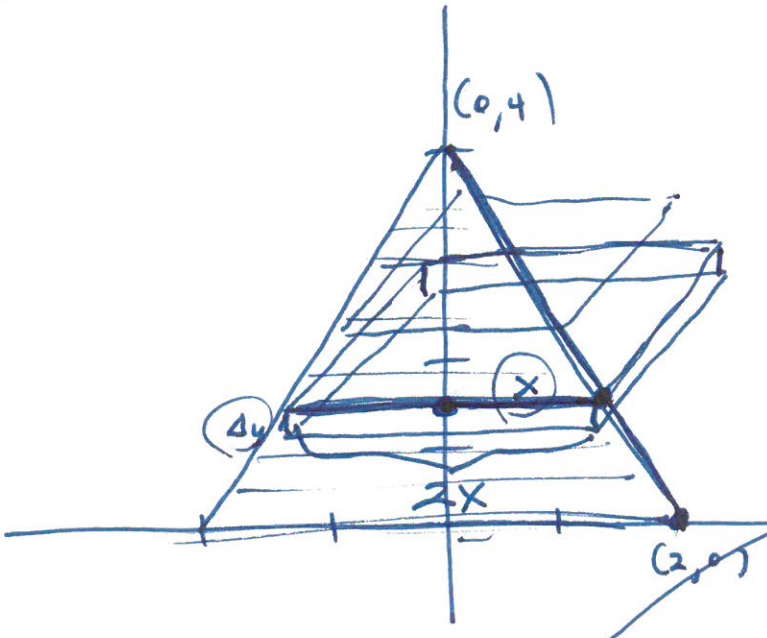
$$V = \pi \left[ \frac{y^{5/3}}{5/3} \right]_1^8$$

$$V = \frac{3\pi}{5} \left[ y^{5/3} \right]_1^8 = \frac{3\pi}{5} \left[ 8^{5/3} - 1^{5/3} \right]$$

$$= \frac{3\pi}{5} \left[ 32 - 1 \right]$$

$$= \frac{3\pi (31)}{5} \approx \_\_\_\_$$




VOL OF SLICE:

$(\text{side})^2 \cdot h$

$(4-y)^2 \cdot dy$

side: $2(x)$ ·

$2\left(\frac{4-y}{2}\right)$

side: $4-y$

LINE: $(2,0)$ & $(0,4)$

$y - y_1 = m(x - x_1)$

$m = \frac{4-0}{0-2} = \frac{4}{-2} = -2$

$= -2$

$y - 4 = -2(x - 0)$

$y - 4 = -2x$

$x = \frac{y-4}{-2}$

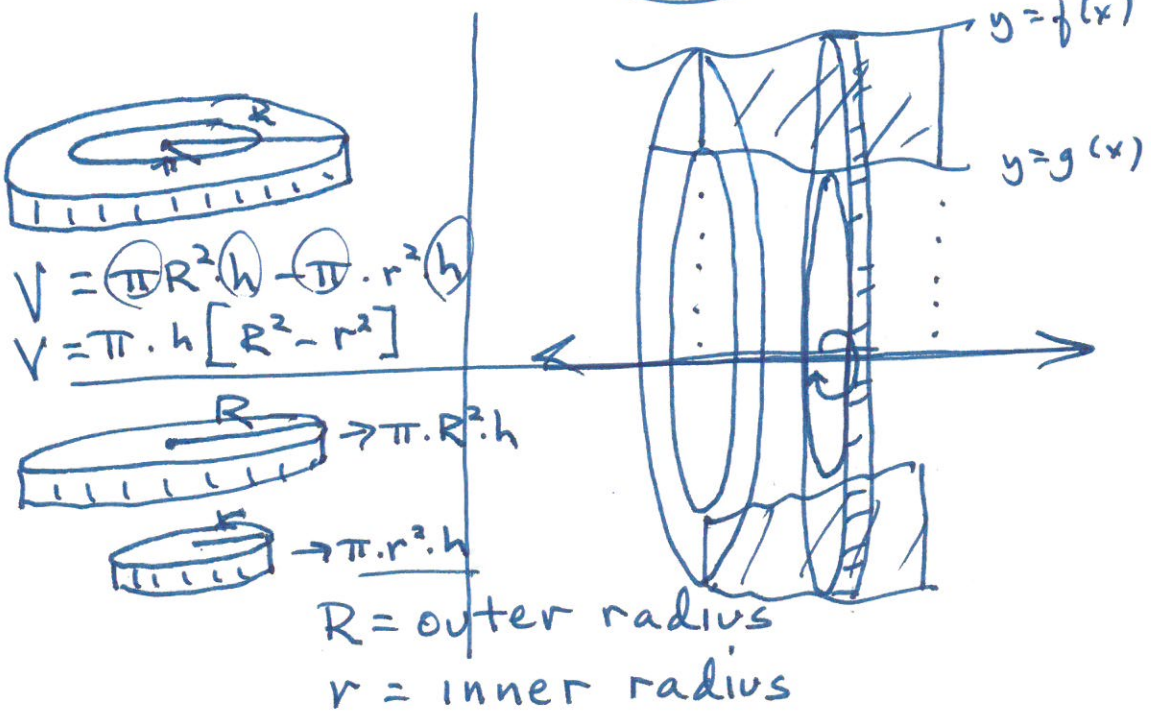
$x = \frac{4-y}{2}$

$$V = \int_0^4 (4-y)^2 \cdot dy$$

$(side)^2 \cdot h$

$$V = \int_0^4 (16 - 8y + y^2)\, dy$$

WASHER: $\int \pi (R^2 - r^2) \cdot h = V$




R = outer radius

r = inner radius