MA141-012

Monday, December 3

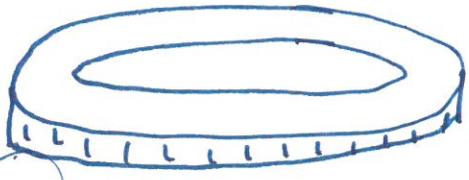
Chapter 5: (today)

5.2:

final exam: MONDAY, DECEMBER 17
6:00 – 9:00 pm SAS 1102

{ ALT: SATURDAY, DECEMBER 15
8:00 – 11:00 am SAS 1102 }

WASHER:



$V = \pi \cdot R^2 \cdot h - \pi \cdot r^2 \cdot h$

$V = \pi (R^2 - r^2) h$

R = outer radius
r = inner radius

80% → 1 pt bonus

85% → 2 pt bonus

90% → 3 pt bonus

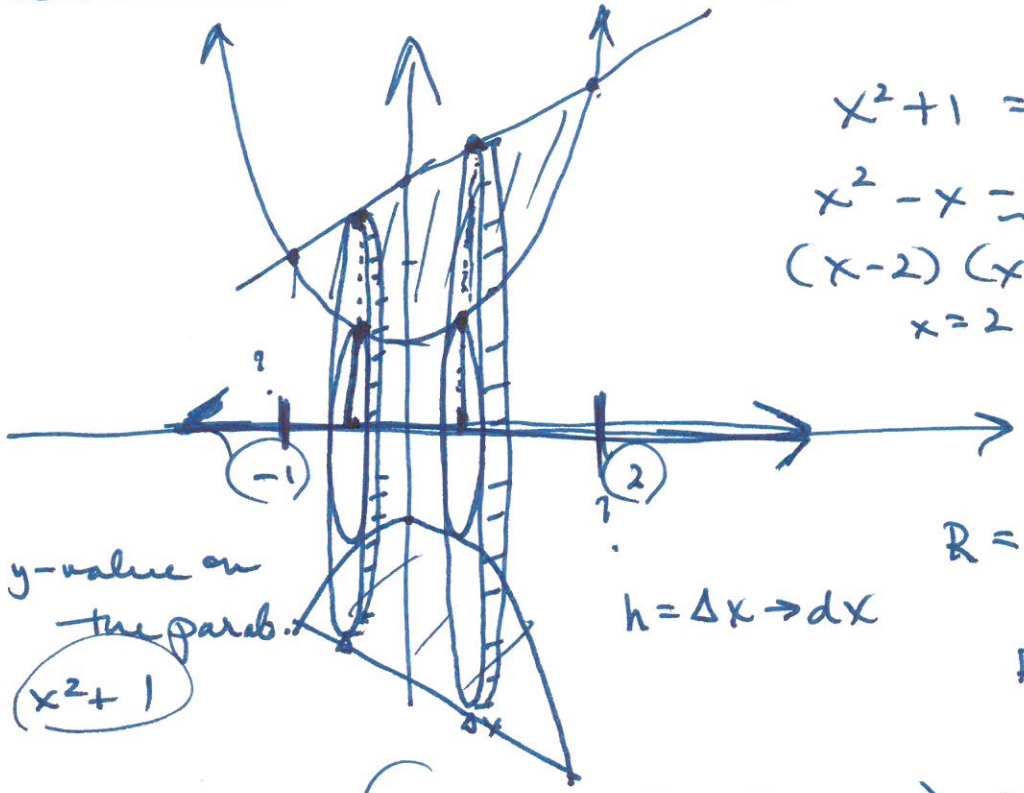
→ 95% → 4 pt. bonus

100% → 5 pt bonus

ex: (washer)

$y = x^2+1$ $\quad y = x+3$

are bounded by these curves revolved about x-axis



$x^2+1 = x+3$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

r = y-value on the parab.

$r = (x^2+1)$

$h = \Delta x \to dx$

R = y-value on the line

$R = (x+3)$

$$V = \int \pi (R^2 - r^2) \cdot h$$

$$V = \pi \int_{-1}^{2} \left[(x+3)^2 - (x^2+1)^2 \right] \cdot dx$$

$$V = \pi \int_{-1}^{2} (x^2 + 6x + 9 - x^4 - 2x^2 - 1)\, dx$$

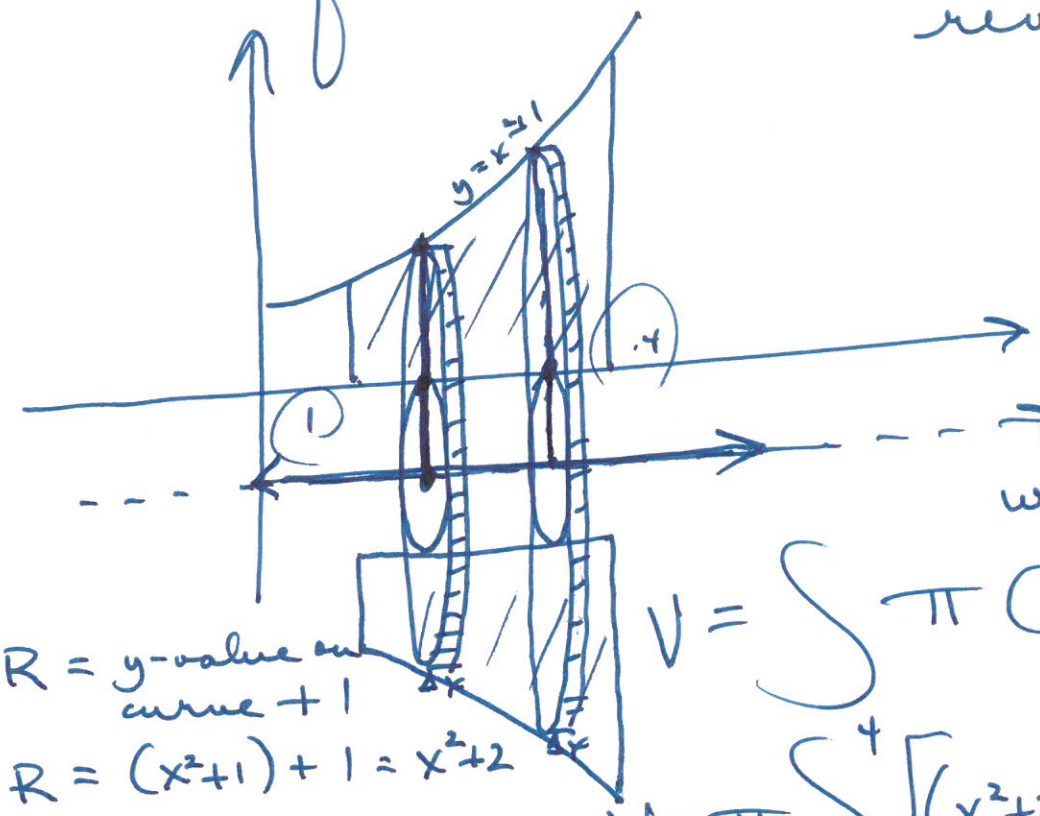
$$V = \pi \int_{-1}^{2} (-x^4 - x^2 + 6x + 8)\, dx$$

$$V = \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} + 6 \cdot \frac{x^2}{2} + 8x \right]_{-1}^{2}$$

$$V = \pi \left[\left(-\frac{(2)^5}{5} - \frac{(2)^3}{3} + 3(2)^2 + 8(2) \right) - \left(-\frac{(-1)^5}{5} - \frac{(-1)^3}{3} + 3(-1)^2 + 8(-1) \right) \right]$$

$$V = \left(\frac{117\pi}{5} \right) \approx \underline{73.5}$$

$y = x^2 + 1$ from $x = 1$ to $x = 4$ revolved about the line $y = -1$



R = y-value on curve $+ 1$

$R = (x^2+1) + 1 = x^2 + 2$

$r = 1$; $h = \Delta x \to dx$

washer:

$$V = \sum \pi (R^2 - r^2) \cdot h$$

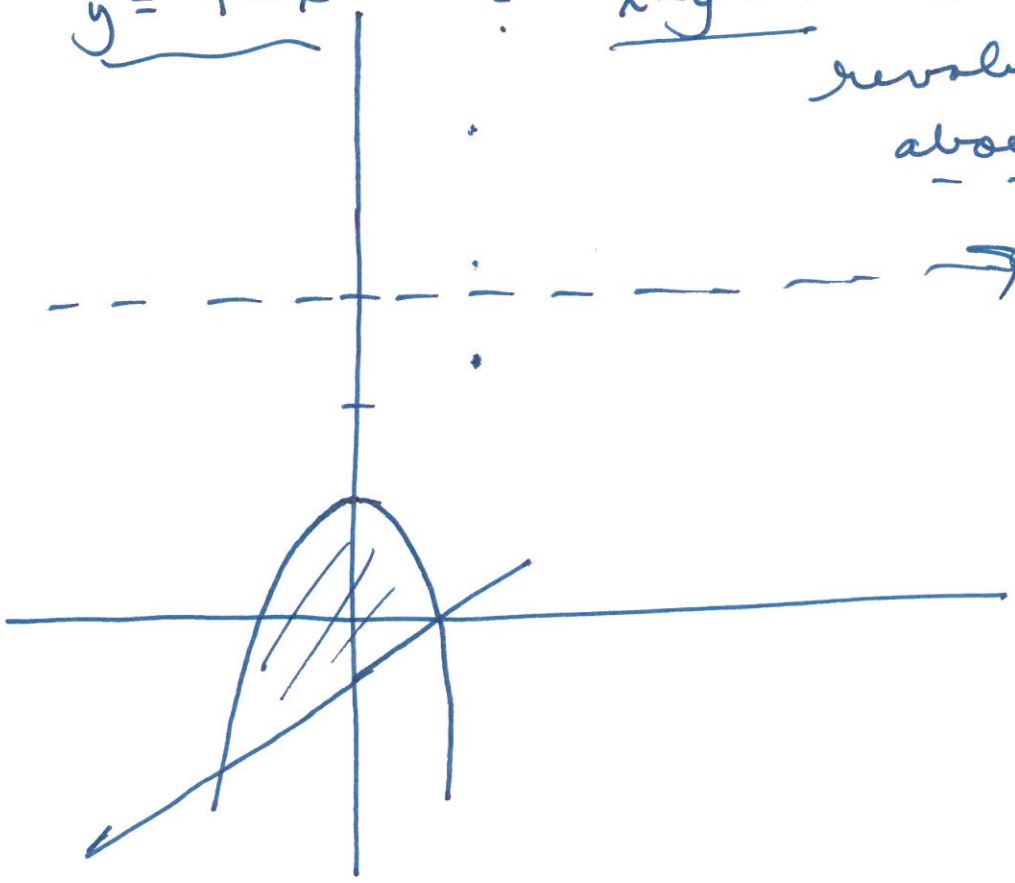
$$V = \pi \int_1^4 \left[(x^2+2)^2 - (1)^2 \right] \cdot dx$$

$$V = \pi \int_1^4 (x^4 + 4x^2 + 4 - 1)\, dx$$

$$V = \pi \int_1^4 (x^4 + 4x^2 + 3)\, dx$$

$$V = \_\_\_\_\_ \pi$$

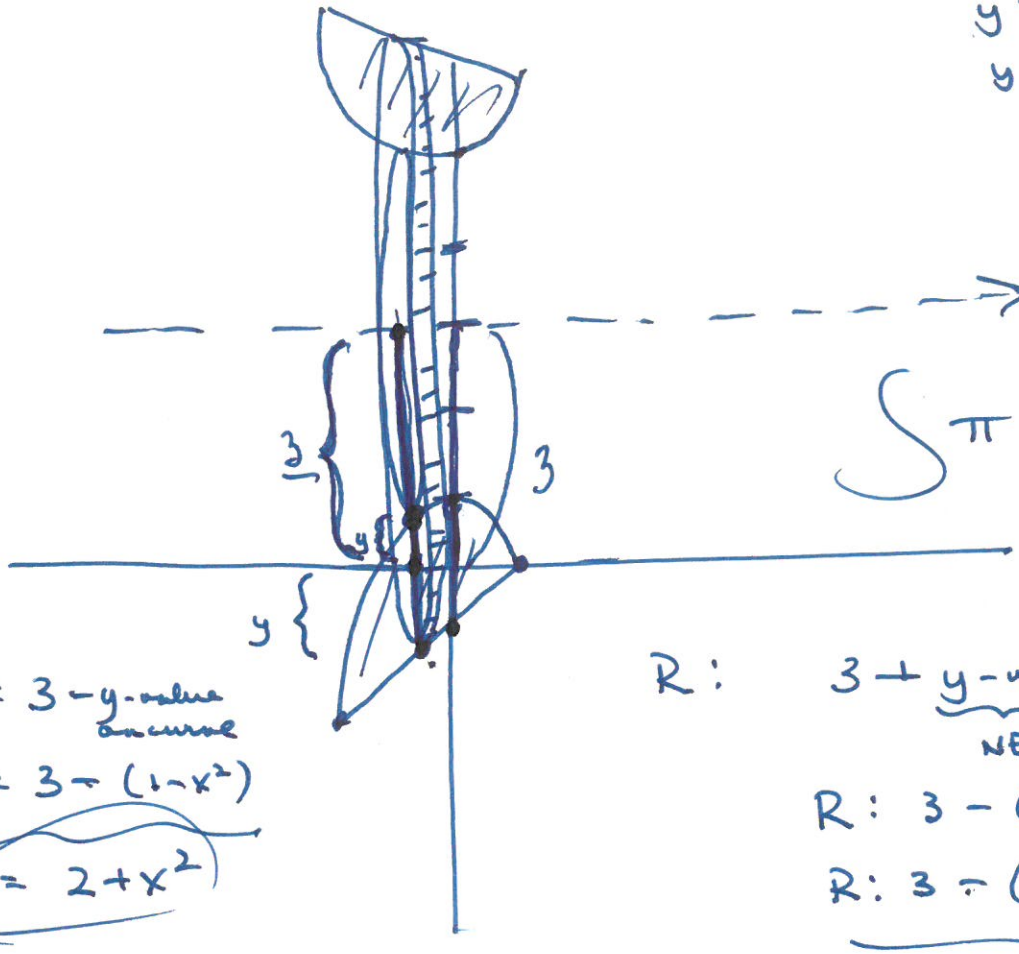
resume: 6:54



$y = 1 - x^2$

$y = x - 1$

(7)



$\int \pi (R^2 - r^2) \cdot h$

$r = 3 - y\text{-value on curve}$

$r = 3 - (1 - x^2)$

$r = 2 + x^2$

R: $3 + \underbrace{y\text{-value on line}}_{\text{NEG}}$

R: $3 - (y\text{-value on line})$

R: $3 - (x - 1) = 4 - x$

$1 - x^2 = x - 1$

solve for x:

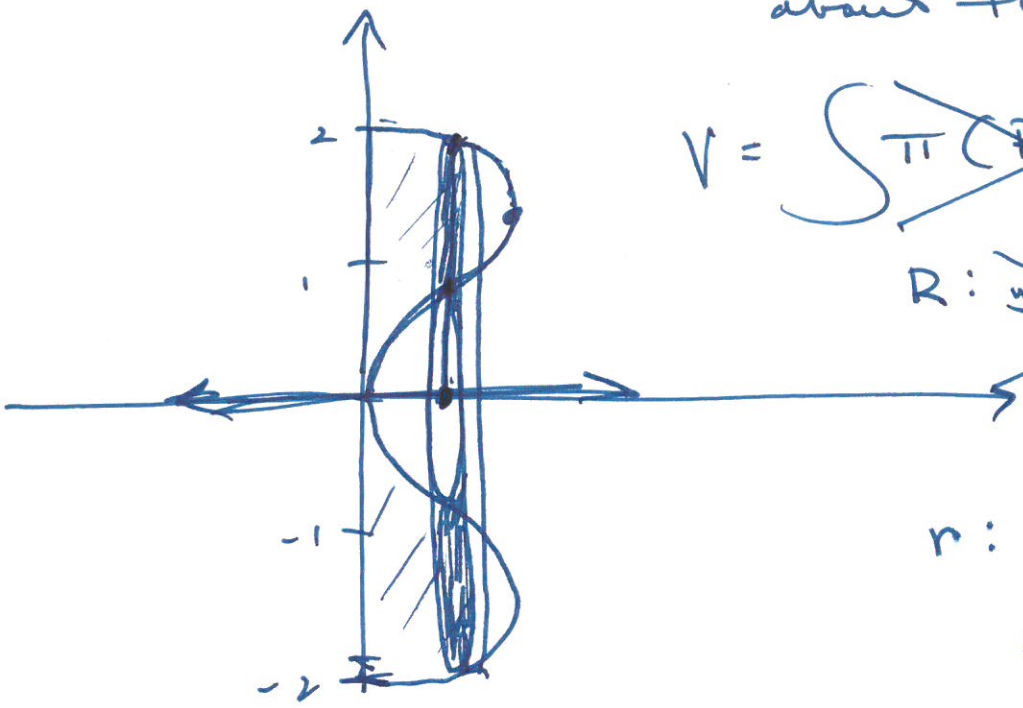
$$\int_{-}^{-} \pi \left((4 - x)^2 - (2 + x^2)^2 \right) \cdot dx$$

5.2:

CYLINDRICAL SHELLS:

$x = 2y^3 - y^4$ ("nose curve")

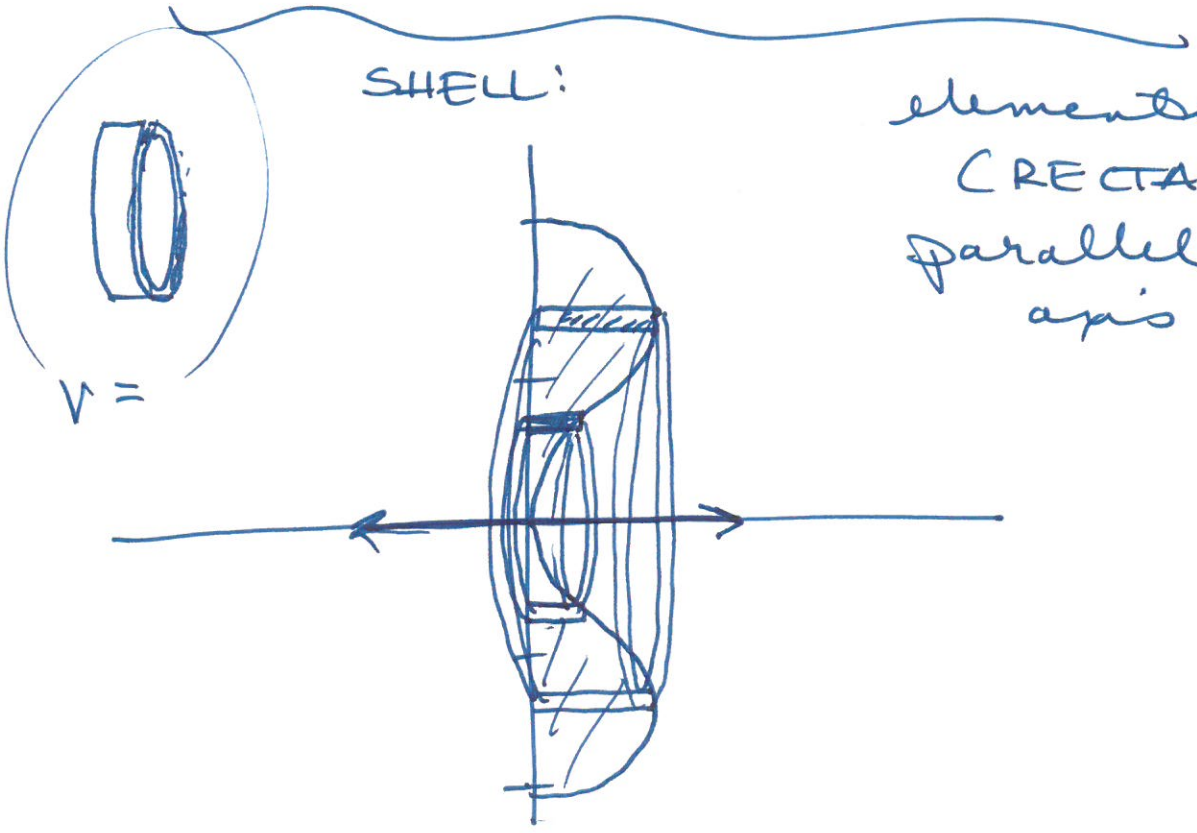
about the x-axis



$V = \int \pi (R^2 - r^2) \cdot h$

R: ~~y-value on curve~~

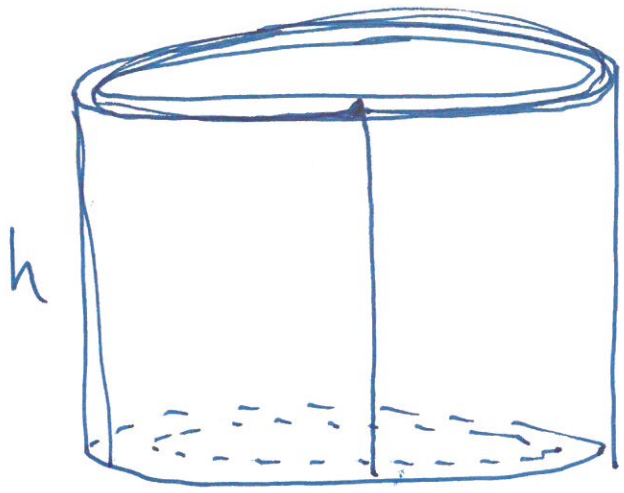
r: ~~y-value on curve~~



SHELL:

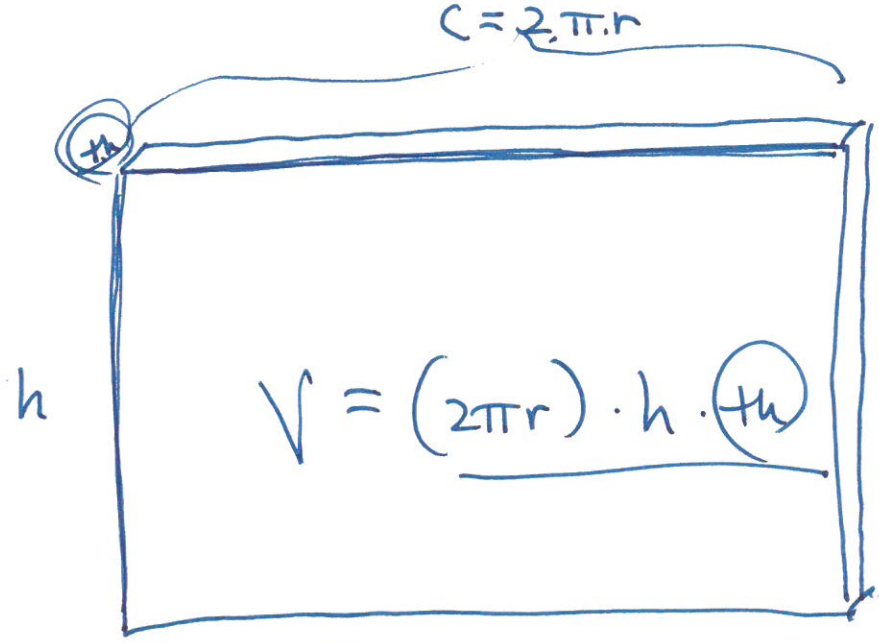
elements of area (RECTANGLES) parallel to the axis of rev.

V =



(th) = thickness
(dy or dx)

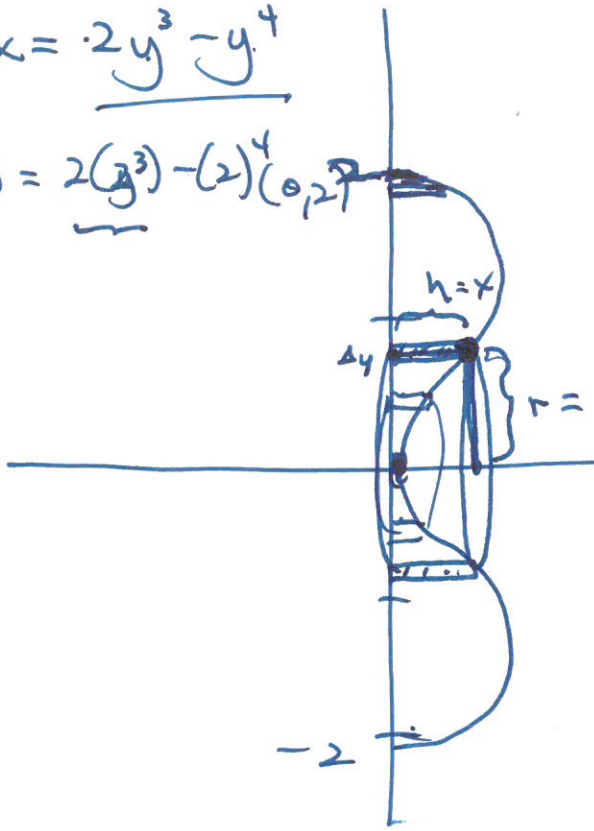
$V =$



$$V = \int 2\pi r \cdot h \cdot (th)$$

$x = 2y^3 - y^4$

$0 = 2(y^3) - (2)^4$ (0, 2)



$V = \int 2\pi (r)(h)(\text{th})$

$\text{th} = \Delta y \to dy$

$r = y$

$h = x = 2y^3 - y^4$

$\text{th} = dy$

$$V = \int_0^2 2\pi \cdot y \cdot (2y^3 - y^4) \cdot dy$$

$$\boxed{V = 2\pi \int_0^2 (2y^4 - y^5)\, dy}$$

set up only - do not integrate nor evaluate