NORTH CAROLINA STATE UNIVERSITY

Department of Mathematics

MA141-012 Test #1 Form A Monday, September 24, 2018 Dr. J. Griggs

Put all work and answers in the stamped blue book provided; one problem per page, please. (the back of a sheet can serve as a new page) Nothing written on the test itself will be graded. Calculators may be used (**not** a graphing calculator nor any calculator that actually **does** calculus). Simplify completely. Turn in this test copy inside of your blue book.

(Seven questions; 14 points each; 2 points for following directions)

1.) a.) For $f(x)=\frac{10}{x^2}$, find $\frac{f(x)-f(a)}{x-a}$ and simplify completely.

b.) Put in standard form; identify; find all relevant points (center, ends of axes, foci, ...) and **graph:**

$$x^2+y^2-6x+2y-15=0$$

2.) **Sketch** the curve represented by the following parametric equations; eliminate the parameter and find the Cartesian (rectangular) equation:

$$x=1+2t \quad and \quad y=6-t^2 \quad -1\le t\le 3$$

3.) Use the formal definition of a limit (ε....δ) to show that $\lim_{x\to 2}(8x-10)=6$.

4.) Find the derivative using the DEFINITION OF DERIVATIVE: $f(x)=\frac{5}{4x+9}$

5.) Evaluate the following limits:

a.) $\lim_{x\to\infty}(\tan^{-1}x)$ b.) $\lim_{x\to 5}\frac{3x^2-15x}{x^2-4x-5}$ c.) $\lim_{x\to 0^+}\ln x$

6.) **Graph** the following: $f(x)=\begin{cases}5 & if\ x\le -2\\ 3x+1 & if\ -2<x<0\\ 7-x^2 & if\ x\ge 0\end{cases}$

For this function, find: $\lim_{x\to 0^-}f(x)=$ $\lim_{x\to 0^+}f(x)=$ $\lim_{x\to 0}f(x)=$

Is this function continuous at x=-2? (verify; 3 possible steps)

7.) Find the average rate of change of $f(x)$ as x changes from x = 1 to x = 4; find the instantaneous rate of change of $f(x)$ at x = 3: $f(x)=6x^2-5x+2$

(use the definition of derivative to find $f'(x)$)

MA141. Test #1

(7 questions; 14 points each).

1.) a.) 7 PTS $f(x)=\frac{10}{x^2}$ find $\frac{f(x)-f(a)}{x-a}$

$$\frac{\frac{10}{x^2}-\frac{10}{a^2}}{x-a}=\frac{\frac{10\cdot a^2}{x^2\cdot a^2}-\frac{10\cdot x^2}{a^2\cdot x^2}}{x-a}=\frac{10a^2-10x^2}{x^2\cdot a^2}\cdot\frac{1}{(x-a)}$$

$$=\frac{10(a^2-x^2)}{x^2\cdot a^2}\cdot\frac{1}{(x-a)}=\frac{10\overset{-1}{\cancel{(a-x)}}(a+x)}{x^2\cdot a^2\cdot\underset{1}{\cancel{(x-a)}}}=\frac{-10(a+x)}{a^2\cdot x^2}$$

$(x\neq a)$

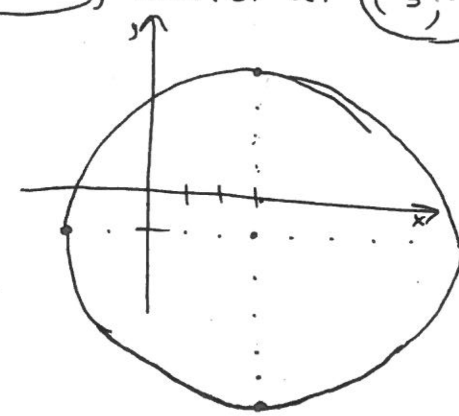
b.) 7 PTS $x^2+y^2-6x+2y-15=0$

$x^2-6x \quad +y^2+2y \quad =15$

complete the square... $\underbrace{x^2-6x+9}+\underbrace{y^2+2y+1}=15+9+1$

$(x-3)^2+(y+1)^2=25$

circle; center at $(3,-1)$; radius $=5$



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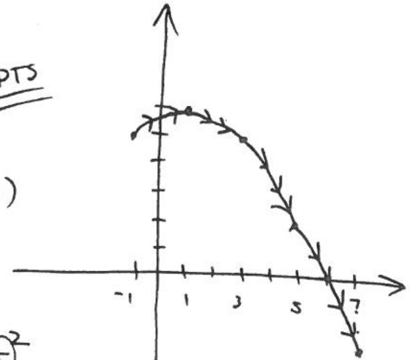
2.) $x = 1+2t \qquad y = 6 - t^2 \qquad -1 \le t \le 3$

t	-1	0	1	2	3
x	-1	1	3	5	7
y	5	6	5	2	-3

7 PTS

$(-1,5)\ldots(1,6)\ldots(3,5)\ldots(5,2)\ldots(7,-3)$



Cartesian Equation:

$x = 1+2t$

$x-1 = 2t$

$\frac{x-1}{2} = t$

subst. for t

$y = 6 - t^2$

$y = 6 - \left[\frac{(x-1)}{2}\right]^2$

or

$y = 6 - \frac{(x^2 - 2x+1)}{4}$

7 PTS

3) $\lim_{x \to 2} (8x-10) = 6$

14 PTS

we want.....
(so we can work backwards)

$|(8x-10)-6| < \epsilon$

$|8x-16| < \epsilon$

$8|x-2| < \epsilon$

$|x-2| < \epsilon/8$

choose $\delta = \epsilon/8$
(or smaller)

* begin with: *

$0 < |x-2| < \delta$

choose $\delta = \epsilon/8$

$|x-2| < \epsilon/8$

$8|x-2| < \epsilon/8\,(8)$

$|8x-16| < \epsilon$

$|(8x-10)-6| < \epsilon$

$\therefore |f(x) - L| < \epsilon$

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4.) $f(x) = \frac{5}{4x+9}$

14 PTS

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{5}{4(x+h)+9} - \frac{5}{4x+9}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{5}{[4(x+h)+9]}\cdot\frac{(4x+9)}{(4x+9)} - \frac{5}{(4x+9)}\cdot\frac{[4(x+h)+9]}{[4(x+h)+9]}}{h}$$

$$= \lim_{h\to 0} \frac{5(4x+9) - 5[4(x+h)+9]}{[4(x+h)+9]\cdot(4x+9)}\cdot\frac{1}{h}$$

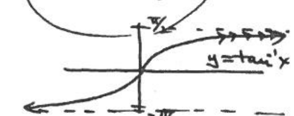
$$= \lim_{h\to 0} \frac{20\cancel{x} + \cancel{45} - 2\cancel{0x} - 20h - \cancel{45}}{[4(x+h)+9]\cdot(4x+9)\cdot h}$$

$$= \lim_{h\to 0} \frac{-20\cancel{h}}{[4(x+h)+9]\cdot(4x+9)\cdot\cancel{h}} \qquad (h \neq 0)$$

$$= \lim_{h\to 0} \frac{-20}{[4(x+\cancel{h})+9]\cdot(4x+9)} = \frac{-20}{(4x+9)^2}$$

5.) a.) $\lim_{x\to\infty} (\tan^{-1} x) = \pi/2$

4 PTS



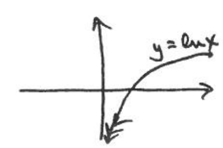
b.) $\lim_{x\to 5} \frac{3(x^2-5x)}{(x-5)(x+1)} = \lim_{x\to 5} \frac{3(x)\cancel{(x-5)}}{\cancel{(x-5)}(x+1)} \quad (x \neq 5)$

6 PTS

$= \lim_{x\to 5} \frac{3x}{x+1} = \frac{15}{6} = \frac{5}{2}$

c.) $\lim_{x\to 0^+} (\ln x) =$ D.N.E. or $-\infty$

4 PTS



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6.) $f(x) = \begin{cases} 5 & x \le -2 \\ 3x+1 & -2 < x < 0 \\ 7-x^2 & x \ge 0 \end{cases}$

$(x \le -2)$	$(-2 < x < 0)$	$(x \ge 0)$
$y = 5$	$y = 3x+1$	$y = 7-x^2$

x	y
-2	5
-3	5
-4	5

constant

x	y
-2	-5
-1	-2
0	1

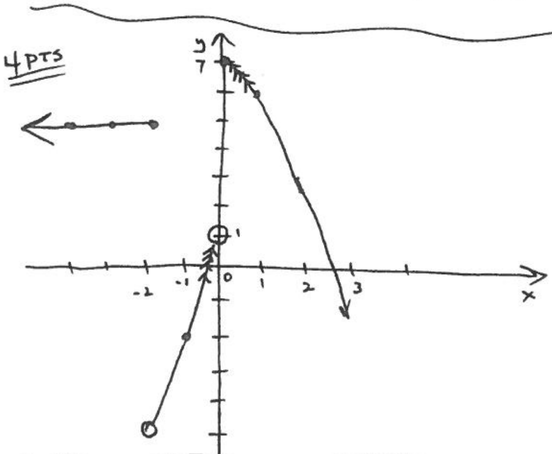
DELETE (-2, -5); DELETE (0, 1)

linear; $m=3$; y-int = 1

x	y
0	7
1	6
2	3

parabola; opens down

4 PTS



$\lim_{x \to 0^-} f(x) = 1$

5 PTS

$\lim_{x \to 0^+} f(x) = 7$

$\lim_{x \to 0} f(x) =$ DOES NOT EXIST

¿ continuous at $x = -2$?

5 PTS

① $f(-2)$ exists? yes, $f(-2) = 5$

② $\lim_{x \to -2} f(x)$ exists? no, $\lim_{x \to -2} f(x)$ D.N.E.

$\left\{ \begin{array}{l} \lim_{x \to -2^+} f(x) = -5 \\ \lim_{x \to -2^-} f(x) = 5 \end{array} \right\}$

$\therefore$ DISCONTINUOUS

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7.) average rate of change; $x=1$ to $x=4$:

6 PTS

$f(x) = 6x^2 - 5x + 2$

$m_{SEC} = \frac{f(4) - f(1)}{4-1} = \frac{78-3}{3} = \frac{75}{3} = 25$

$f(4) = 6(4)^2 - 5(4) + 2 = 96 - 20 + 2 = 78$

$f(1) = 6(1)^2 - 5(1) + 2 = 6 - 5 + 2 = 3$

instantaneous rate of change at $x = 3$

$f(x) = 6x^2 - 5x + 2$

$f'(x) = 12x - 5$ (by DEF OF DERIV ~~OR SHORTCUT~~)

$f'(3) = 12(3) - 5 = 36 - 5 = 31$