NORTH CAROLINA STATE UNIVERSITY

Department of Mathematics

MA141-012 Test #1 Form B Monday, September 24, 2018 Dr. J. Griggs

Put all work and answers in the stamped blue book provided; one problem per page please. (the back of a sheet can serve as a new page) Nothing written on the test itself will be graded. Calculators may be used (**not** a graphing calculator nor any calculator that actually **does** calculus). Simplify completely. Turn in this test copy inside of your blue book.

(Seven questions; 14 points each; 2 points for following directions)

1.) a.) For $f(x)=\frac{5}{x^2}$, find $\frac{f(x)-f(a)}{x-a}$ and simplify completely.

b.) Put in standard form; identify; find all relevant points (center, ends of axes, foci, ...) and **graph:**

$$x^2+y^2-8x+2y+8=0$$

2.) **Sketch** the curve represented by the following parametric equations; eliminate the parameter and find the Cartesian (rectangular) equation:

$$x=1+3t \quad and \quad y=2-t^2 \qquad -1\le t\le 3$$

3.) Use the formal definition of a limit $(\varepsilon....\delta)$ to show that $\lim_{x\to 2}(3x+5)=11$.

4.) Find the derivative using the DEFINITION OF DERIVATIVE: $f(x)=\frac{4}{5x+7}$

5.) Evaluate the following limits:

a.) $\lim_{x\to\infty}(\tan^{-1}x)$ b.) $\lim_{x\to 5}\frac{2x^2-10x}{x^2-4x-5}$ c.) $\lim_{x\to 0^+}\ln x$

6.) **Graph** the following: $f(x)=\begin{cases}3 & if\ x\le -2\\ 2x+1 & if\ -2<x<0\\ 4-x^2 & if\ x\ge 0\end{cases}$

For this function, find: $\lim_{x\to 0^-}f(x)=$ $\lim_{x\to 0^+}f(x)=$ $\lim_{x\to 0}f(x)=$

Is this function continuous at x=-2? (verify; 3 possible steps)

7.) Find the average rate of change of $f(x)$ as x changes from x = 1 to x = 4; find the instantaneous rate of change of $f(x)$ at x = 3: $f(x)=5x^2-6x+2$

(use the definition of derivative to find $f'(x)$)

MA 141 TEST # 1

(7 questions; 14 points each)

1. a.) 7 pts

$f(x)=\frac{5}{x^2}$ $\qquad \frac{f(x)-f(a)}{x-a}=\frac{\frac{5}{x^2}-\frac{5}{a^2}}{x-a}$

$$=\frac{\frac{5}{x^2}\cdot\frac{a^2}{a^2}-\frac{5}{a^2}\cdot\frac{x^2}{x^2}}{(x-a)}=\frac{5a^2-5x^2}{x^2\cdot a^2}\cdot\frac{1}{(x-a)}$$

$$=\frac{5(a^2-x^2)}{x^2\cdot a^2}\cdot\frac{1}{(x-a)}=\frac{5\overset{-1}{\cancel{(a-x)}}(a+x)}{x^2a^2\underset{1}{\cancel{(x-a)}}}=\frac{-5(a+x)}{a^2x^2}$$

$(x\ne a)$

b.) $x^2+y^2-8x+2y+8=0$

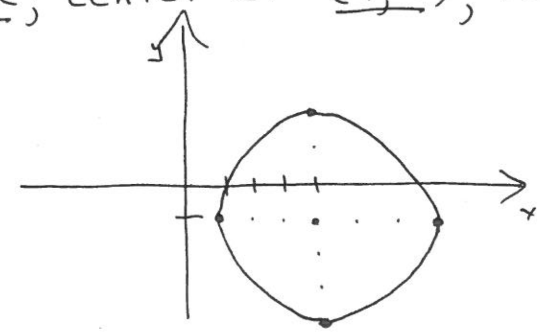
7 pts

$x^2+8x \quad +y^2+2y \quad =-8$

$\underbrace{x^2-8x+16}+\underbrace{y^2+2y+1}=-8+16+1$

$(x-4)^2+(y+1)^2=9$

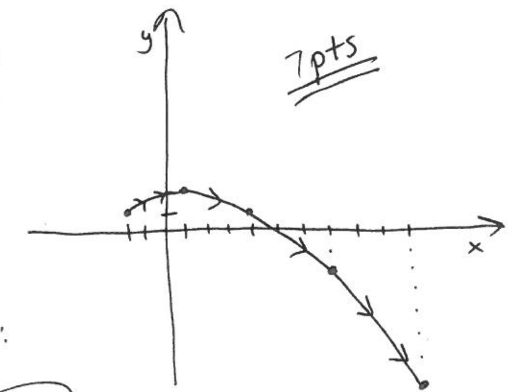
circle; center at $(4,-1)$; radius $=3$



② $x = 1+3t \qquad y = 2 - t^2 \qquad -1 \le t \le 3$

t	-1	0	1	2	3
x	-2	1	4	7	10
y	1	2	1	-2	-7

$(-2,1)\ldots(1,2)\ldots(4,1)\ldots(7,-2)$
$\ldots(10,-7)$



7pts

Cartesian Equation:

$x = 1+3t$

$x-1 = 3t$

$\dfrac{x-1}{3} = t$

$y = 2 - t^2$

$y = 2 - \left[\dfrac{(x-1)}{3}\right]^2$

or

$y = 2 - \dfrac{(x^2-2x+1)}{9}$

7pts

③ $\lim\limits_{x \to 2} (3x+5) = 11$

14pts

we want ....

$|(3x+5)-11| < \epsilon$

$|3x-6| < \epsilon$

$3\,|x-2| < \epsilon$

$|x-2| < \epsilon/3$

choose $\delta = \epsilon/3$
(or smaller)

begin with...

$0 < |x-2| < \delta$

$\boxed{\text{choose } \delta = \epsilon/3}$

$|x-2| < \epsilon/3$

$3|x-2| < 3(\epsilon/3)$

$|3x-6| < \epsilon$

$|3x+5-5-6| < \epsilon$

$|(3x+5)-11| < \epsilon$

$\therefore\ |f(x)-L| < \epsilon$

④ $f(x) = \dfrac{4}{5x+7}$

14pts

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{4}{5(x+h)+7} - \dfrac{4}{5x+7}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{4}{5(x+h)+7}\cdot\dfrac{(5x+7)}{(5x+7)} - \dfrac{4}{5x+7}\cdot\dfrac{[5(x+h)+7]}{[5(x+h)+7]}}{h}$$

$$= \lim_{h\to 0} \frac{4(5x+7) - 4[5(x+h)+7]}{[5(x+h)+7]\cdot(5x+7)} \cdot \frac{1}{h}$$

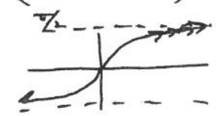
$$= \lim_{h\to 0} \frac{\cancel{20x} + \cancel{28} - \cancel{20x} - 20h - \cancel{28}}{[5(x+h)+7]\cdot[5x+7]\cdot h}$$

$$= \lim_{h\to 0} \frac{-20\cdot \cancel{h}}{[5(x+h)+7]\cdot[5x+7]\cdot\cancel{h}} \qquad (h \ne 0)$$

$$= \lim_{h\to 0} \frac{-20}{[5(x+\cancel{h})+7]\cdot[5x+7]} = \frac{-20}{(5x+7)^2}$$

⑤ a.) $\lim\limits_{x\to\infty} (\tan^{-1} x) = \pi/2$

4pts



b.) $\lim\limits_{x\to 5} \dfrac{2(x^2-5x)}{(x-5)(x+1)} = \lim\limits_{x\to 5} \dfrac{2\cdot x\cancel{(x-5)}}{\cancel{(x-5)}(x+1)} \quad (x \ne 5)$

6pts

$= \lim\limits_{x\to 5} \dfrac{2x}{x+1} = \dfrac{10}{6} = \dfrac{5}{3}$

c.) $\lim\limits_{x\to 0^+} (\ln x) =$ D.N.E. or $-\infty$

4pts

⑥ $f(x) = \begin{cases} 3 & x \le -2 \\ 2x+1 & -2 < x < 0 \\ 4 - x^2 & x \ge 0 \end{cases}$

$(x \le -2)$
$y = 3$

x	y
-2	3
-3	3
-4	3

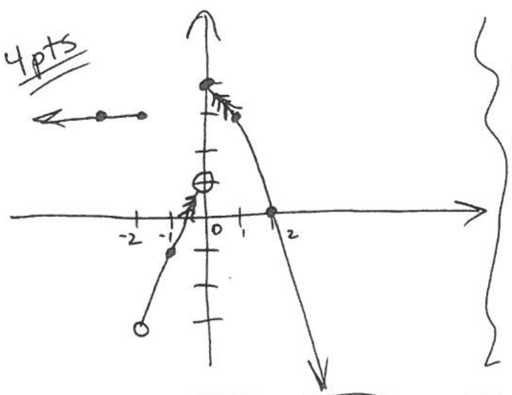
$(-2 < x < 0)$
$y = 2x + 1$

x	y	
-2	-3	delete
-1	-1	
0	1	delete

$(x \ge 0)$
$y = 4 - x^2$

x	y
0	4
1	3
2	0

4pts



$\lim_{x \to 0^-} f(x) = 1$

$\lim_{x \to 0^+} f(x) = 4$

5pts

$\lim_{x \to 0} f(x) =$ DOES NOT EXIST

¿ continuous at $x = -2$?

5pts

① ¿ $f(-2)$ exists? yes, $f(-2) = 3$

② ¿ $\lim_{x \to -2} f(x)$ exists? no, $\lim_{x \to -2} f(x)$ D.N.E.

$\left(\begin{array}{l} \lim_{x \to -2^+} f(x) = -3 \\ \lim_{x \to -2^-} f(x) = 3 \end{array} \right)$

∴ DISCONTIN

⑦ average rate of change; $x = 1 \to x = 4$

6pts $f(x) = 5x^2 - 6x + 2$

$m_{SEC} = \dfrac{f(4) - f(1)}{4 - 1} = \dfrac{58 - 1}{3} = \dfrac{57}{3} = 19$

$f(4) = 5 \cdot (4)^2 - 6(4) + 2 = 80 - 24 + 2 = 58$

$f(1) = 5 \cdot (1)^2 - 6(1) + 2 = 1$

instantaneous rate of change; at $x = 3$

8pts $m_{TAN} = f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{(5(x+h)^2 - 6(x+h) + 2) - (5x^2 - 6x + 2)}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{6x} - 6h + \cancel{2} - \cancel{5x^2} + \cancel{6x} - \cancel{2}}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{h}(10x + 5h - 6)}{\cancel{h}} \quad (h \neq 0)$

$= \lim_{h \to 0} (10x + \underbrace{5h}_{0} - 6) = 10x - 6$

at $x = 3$ ....

$f'(3) = 10(3) - 6 = 24$