NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA141-012 Test #2A Wednesday, October 10, 2018 Dr. J. Griggs

Please put all work and answers in the stamped blue/green book provided. No graphing calculators; do **not** use a calculator that does calculus. Validate all work with appropriate calculus techniques. Simplify completely – unless directed otherwise in the problem. Fold the test copy and turn it in along with the blue/green book. (Name; row/seat on the front)

1.) Find $\frac{dy}{dx}$:

a.) $y^4 + 3xy = 5x^2 + xy^2$ b.) $y = e^x \cdot \cot x + \csc^4 x$

2.) Find y' :

a.) $y = \sqrt{1-x^2} \cdot \sin^{-1} x$ b.) $y = \frac{1-\sec x}{\tan x}$

3.) Compute the first and second derivatives of $f(x) = \frac{8x-1}{5x+3}$

4.) A ball is projected upward from a platform 30 feet above the ground with an initial velocity of 56 feet per second. The height of the ball at time t (in seconds) is given by $s(t) = -16t^2 + 56t + 30$. Find the velocity and acceleration of the ball as functions of time. How long is the ball in the air? What is the velocity of the ball when it hits the ground?

5.) Find $f'(x)$: a.) $f(x) = \left(\frac{x^2+17}{2x^2+x+1}\right)^3$ b.) $f(x) = \arctan(\sin(x^2+3x))$

6.) Find the equation of the line tangent to the curve $f(x) = \sin x \cdot \cos x$ at the point $\left(\frac{\pi}{4}, \frac{1}{2}\right)$.

7.) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$: $4x^2 + 9y^2 = 36$

MA 141 TEST #2A J. GRIGGS

SOLUTIONS

(7 QUESTIONS — 14 POINTS EACH)

1.) a.) $y^4 + \underbrace{3xy} = 5x^2 + \underbrace{xy^2}$

7pts

DERIV: $4 \cdot y^3 \cdot \frac{dy}{dx} + \left[(3x) \cdot \frac{dy}{dx} + (y) \cdot (3)\right] = 10x + \left[(x) \cdot 2y\frac{dy}{dx} + y^2(1\right.$

$$4y^3 \frac{dy}{dx} + 3x\frac{dy}{dx} - 2xy\frac{dy}{dx} = 10x + y^2 - 3y$$

$$\frac{dy}{dx}\left[4y^3 + 3x - 2xy\right] = 10x + y^2 - 3y$$

$$\frac{dy}{dx} = \frac{10x + y^2 - 3y}{4y^3 + 3x - 2xy}$$

b.) $y = e^x \cdot \cot x + (\csc x)^4$

7pts

DERIV: $\frac{dy}{dx} = \left(e^x \cdot \left[-\csc^2 x\right] + (\cot x) \cdot e^x\right) + 4(\csc x)^3 \cdot (-\csc x \cdot \cot x)$

$$\frac{dy}{dx} = e^x\left[\cot x - \csc^2 x\right] - 4(\csc x)^4 (\cot x)$$

2.) a.) $y = (1 - x^2)^{1/2} \cdot \sin^{-1} x$

7pts

$y' = \cancel{(1-x^2)^{1/2}} \cdot \left[\frac{1}{\cancel{\sqrt{1-x^2}}}\right] + (\sin^{-1} x) \cdot \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$$y' = 1 - \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}}$$

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2.) b.) $y = \dfrac{1-\sec x}{\tan x}$

7 pts

$$y' = \frac{(\tan x)(-1\cdot\sec x\tan x) - (1-\sec x)(\sec^2 x)}{(\tan x)^2}$$

$$y' = \frac{-\sec x\cdot\tan^2 x - \sec^2 x + \sec^3 x}{(\tan x)^2}$$

(ok to stop here)

$$y' = \frac{+\sec x\,(-\tan^2 x - \sec x + \sec^2 x)}{(\tan x)^2}$$

$$y' = \frac{+\sec x\,(\underbrace{-\tan^2 x + \sec^2 x}_{} - \sec x)}{(\tan x)^2}$$

or

$$y' = \frac{+\sec x\,(1-\sec x)}{(\tan x)^2}$$

3.) $f(x) = \dfrac{8x-1}{5x+3}$ find $f'(x)$ & $f''(x)$:

14 pts

$$f'(x) = \frac{(5x+3)(8)-(8x-1)(5)}{(5x+3)^2} = \frac{\cancel{40x}+24-\cancel{40x}+5}{(5x+3)^2}$$

$$f'(x) = \frac{29}{(5x+3)^2} \quad \text{or} \quad f'(x) = 29(5x+3)^{-2}$$

$$f''(x) = 29\left[-2(5x+3)^{-3}\cdot(5)\right] = \frac{-290}{(5x+3)^3} = f''(x)$$

4.) $s(t) = -16t^2 + 56t + 30$

14pts

$s(t)$: FEET

t : SECONDS

$s'(t) = \boxed{v(t) = -32t + 56}$

$s''(t) = v'(t) = \boxed{a(t) = -32}$

in the air for how long:
(when does it hit the ground!)

$0 = s(t) = -16t^2 + 56t + 30$

$$t = \frac{-56 \pm \sqrt{(56)^2 - 4(-16)(30)}}{2(-16)} =$$

$$t = \frac{-56 \pm \sqrt{3136 + 1920}}{-32} = \frac{-56 \pm 71.106}{-32}$$

$$\begin{cases} t_1 = \dfrac{-56 - 71.106}{-32} \approx \boxed{3.97 \text{ sec}} \\ t_2 = \cancel{\dfrac{-56 + 71.106}{-32} \approx \text{NEG}} \ (\text{time} \geq 0) \end{cases}$$

$$v(3.97) = -32(3.97) + 56 \approx \boxed{-71.04 \ \frac{\text{FT}}{\text{SEC}}}$$

(when it hits the ground)

5.) a.) $f(x) = \left(\dfrac{x^2+17}{2x^2+x+1}\right)^3$

7pts

$$f'(x) = 3\left(\frac{x^2+17}{2x^2+x+1}\right)^2 \cdot \left[\frac{(2x^2+x+1)(2x) - (x^2+17)(4x+1)}{(2x^2+x+1)^2}\right]$$

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$$f'(x) = 3\,\frac{(x^2+17)^2}{(2x^2+x+1)^2}\left[\frac{\cancel{4x^3}+2x^2+2x-\cancel{4x^3}-68x-x^2-17}{(2x^2+x+1)^2}\right]$$

$$f'(x) = \frac{3\,(x^2+17)^2\,[x^2-66x-17]}{(2x^2+x+1)^4}$$

b.) $f(x) = \arctan(\sin(x^2+3x))$

7pts

$$f'(x) = \frac{1}{1+(\sin(x^2+3x))^2}\cdot[\cos(x^2+3x)]\cdot(2x+3)$$

$$f'(x) = \frac{(2x+3)\cdot\cos(x^2+3x)}{1+[\sin(x^2+3x)]^2}$$

6.) $y - \frac{1}{2} = m_{TAN}\left(x - \pi/4\right)$ $\quad(\pi/4, 1/2)$

14 pts

$m_{TAN} = f'(x) = (\sin x)\cdot(-\sin x) + (\cos x)(\cos x)$

$f'(x) = -(\sin x)^2 + (\cos x)^2$

$f'(\pi/4) = -(\sin\frac{\pi}{4})^2 + (\cos\pi/4)^2$

$f'(\pi/4) = -\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 0$

$y - \frac{1}{2} = 0\,(x - \pi/4)$

$y - \frac{1}{2} = 0$

$y = 1/2$

7.) find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$:

$$4x^2 + 9y^2 = 36$$

DERIV: $8x + 18y \cdot \frac{dy}{dx} = 0$

$$18y \cdot \frac{dy}{dx} = -8x$$

$$\frac{dy}{dx} = \frac{-8x}{18y} = \frac{-4x}{9y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-4x}{9y}$$

DERIV: $\frac{d^2y}{dx^2} = \frac{(9y)\cdot(-4) - (-4x)\cdot 9 \cdot \frac{dy}{dx}}{[9y]^2}$

$$\frac{d^2y}{dx^2} = \frac{-36y + 36x\left(\frac{dy}{dx}\right)}{[9y]^2} = \frac{-36y + 36x\left(\frac{-4x}{9y}\right)}{81y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-36y - \frac{144x^2}{9y}}{81y^2} = \frac{-36y - \frac{16x^2}{y}}{81y^2} = \frac{d^2y}{dx^2}$$

OK to STOP HERE

or

$$\frac{d^2y}{dx^2} = \frac{-36y^2 - 16x^2}{81y^3} = \frac{-4[9y^2 + 4x^2]}{81y^3} = \frac{-4(36)}{81y^3} = \frac{-16}{9y^3}$$