NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA141-012 Test #3 Form A Wednesday, October 31, 2018 Dr. J. Griggs

Please put all work and answers on the blue/green book provided. Fold the test and turn it in with the blue/green book. Do **not** use a graphing calculator nor any other calculator that does calculus. Validate all work and graphs with appropriate calculus techniques. Put your name and row number-seat number on the front of the blue book.

1.) Evaluate: a.) $Find\ \frac{dy}{dx}$: $y = (\ln x)^{\sin x}$ (use log. diff.) b.) $\lim_{x \to 0^+} x^{x^2}$

2.) For the function $f(x) = x^3 - x^2 - x + 4$ on [-2, 3], using the first and second derivative - find the critical points and points of inflection; where the function is increasing, decreasing; where it is concave up, concave down; all relative and absolute maximum and minimum values; graph the function.

3.) Use Newton's Method to find the positive root of the equation $f(x) = 2x - e^{-x} = 0$ using $x_1 = .3$, find x_2 *and* x_3 .

4.) A closed wooden box (top included) is to be constructed to have a volume of 45 cubic feet. The length of the box must be 5 times the height. Once it is built, it will be coated with a very expensive acrylic – so its surface area needs to be minimized. What dimensions should the box have in order to minimize the surface area?

5.) A balloon is rising at a constant speed of 6 ft/sec. A boy is biking along a straight road at a speed of 18 ft/sec. When he passes under the balloon, it is 48 ft above him. How fast is the distance between the boy and the balloon changing 2 seconds later?

6.) Using $f(x) = x^2 - 4x + 2$ on the interval $[1,4]$, verify the hypotheses of the Mean Value Theorem and find c in $[1,4]$ guaranteed by the conclusion. Construct a graph/sketch of this scenario.

MA141-012 TEST #3 A, B 10/31/18

(6 QUESTIONS; 16 POINTS EACH)

1.) a.) $y = (\ln x)^{\sin x}$

8pts

$$\ln y = \ln\left[(\ln x)^{\sin x}\right]$$

$$\ln y = (\sin x)\cdot[\ln(\ln x)]$$

DERIV....

$$\frac{1}{y}\cdot\frac{dy}{dx} = (\sin x)\cdot\left[\frac{1}{\ln x}\cdot\frac{1}{x}\right] + [\ln(\ln x)]\cdot\cos x$$

$$\frac{dy}{dx} = y\left[\frac{\sin x}{x\cdot\ln x} + (\cos x)(\ln(\ln x))\right]$$

$$\frac{dy}{dx} = (\ln x)^{\sin x}\left[\frac{\sin x}{x\cdot\ln x} + (\cos x)(\ln(\ln x))\right]$$

b.) $\lim\limits_{x\to 0^+} x^{x^2}$

8pts

(rewrite) $\lim\limits_{x\to 0^+} e^{\ln x^{x^2}}$

$$\lim_{x\to 0^+} e^{(x^2)\cdot(\ln x)}$$

$$e^{\boxed{\lim\limits_{x\to 0^+} x^2\cdot\ln x}} \; *$$

(rewrite)

$$\lim_{x\to 0^+} \frac{\ln x \to -\infty}{\frac{1}{x^2} \to \infty} \xrightarrow{\text{L'HOP}}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+}\left[\frac{1}{x}\cdot\frac{x^3}{-2}\right]$$

$$= \lim_{x \to 0^+} \frac{x^2}{-2} = 0$$

$$* \therefore e^0 = 1$$

2.) $f(x) = x^3 - x^2 - x + 4$ on $[-2, 3]$

16pts

endpoints:

$f(-2) = (-2)^3 - (-2)^2 - (-2) + 4 = -6 \rightarrow (-2, -6)$

$f(3) = (3)^3 - (3)^2 - (3) + 4 = 19 \rightarrow (3, 19)$

$f'(x) = 3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3} \quad x = 1$

$f(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 4 = \frac{113}{27} \approx 4.185$

$f(1) = 1^3 - 1^2 - 1 + 4 = 3$

critical points: $(-\frac{1}{3}, 4.185)$ & $(1, 3)$ "FLAT"

$f'(x)$:

$f'(-1) = +$ | $f'(0) = -$ | $f'(2) = +$

$f(x)$ INCR $-\frac{1}{3}$ $f(x)$ DECR 1 $f(x)$ INCR

($f'(x)$ is never undef.)

$f''(x) = 6x - 2 = 0$

$6x = 2 \quad x = 1/3$

$f(1/3) = (1/3)^3 - (1/3)^2 - (1/3) + 4 = \frac{97}{27} \approx 3.59$

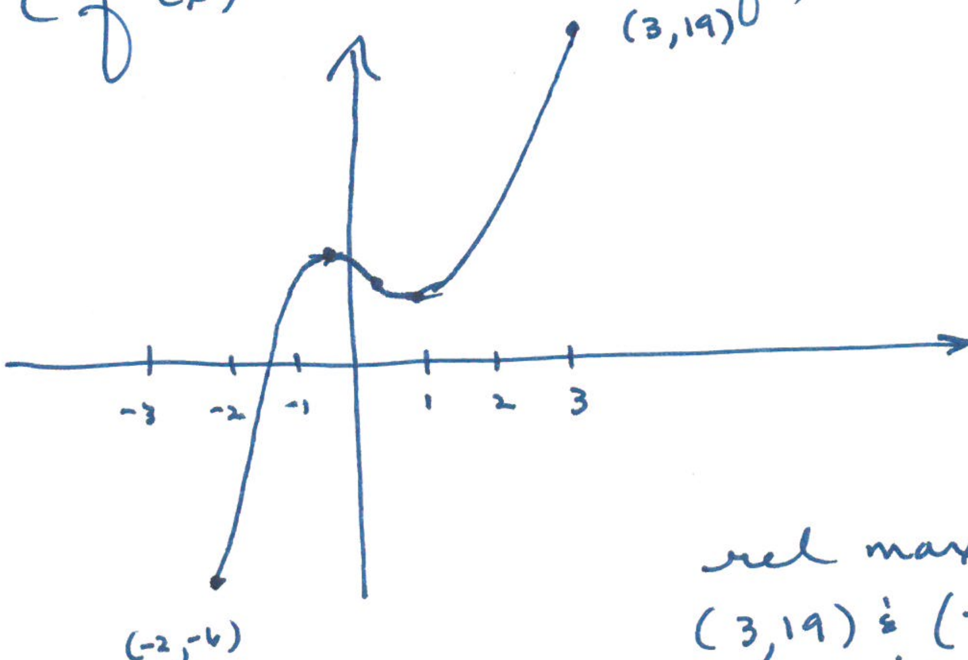
$(1/3, 3.59)$ point of inflection

$f''(x)$:

$f''(0) = -$ | $f''(1) = +$

$f(x)$ CONC. DOWN, $1/3$, $f(x)$ CONC. UP

($f''(x)$ is never undef.)



rel max:

$(3,19)$ & $(-1/3, 4.185)$

rel min:

$(1,3)$ & $(-2,-6)$

ABS MAX: 19 $(3,19)$

ABS MAX: −6 $(-2,-6)$

3.) $f(x) = 2x - e^{-x}$ $\quad x_1 = .3$

16 pts

$$f'(x) = 2 + e^{-x}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = .3 - \frac{f(.3)}{f'(.3)}$$

$$x_2 = .3 - \frac{[.6 - .7408]}{[2 + .7404]}$$

$$x_2 = .3 + .0514 = \underline{\underline{.3514}}$$

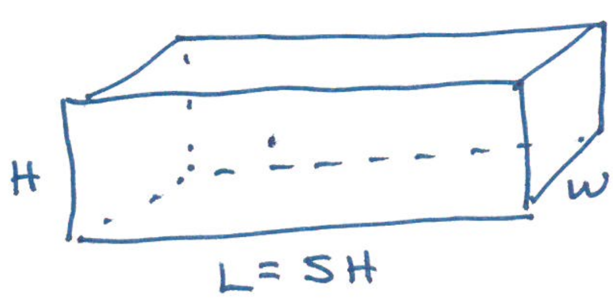
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = .3514 - \frac{[.7028 - .7037]}{[2 + .7037]}$$

$$x_3 = .3514 + .00033$$

$$x_3 = \underline{\underline{.35173}}$$

4.)

16 pts



$$V = 45$$

$$(H)(5H)(w) = 45$$

$$5H^2 w = 45$$

$$w = \frac{45}{5H^2} = \frac{9}{H^2}$$

(page 5)

$$S = 2\left[(H)(5H)\right] + 2\left[(w)(H)\right] + 2\left[(5H)(w)\right]$$

$$S = 10H^2 + 2wH + 10wH$$

$$S = 10H^2 + 12wH \qquad \text{subst.} \qquad w = \frac{9}{H^2}$$

$$S = 10H^2 + 12\left(\frac{9}{H^2}\right)\cdot H = 10H^2 + \frac{108}{H}$$

$$S' = 20H - \frac{108}{H^2} = 0$$

$$\frac{20H}{1} = \frac{108}{H^2} \qquad 108 = 20H^3$$

$$H^3 = \frac{108}{20} = \frac{54}{10} = 5.4$$

$$H = \sqrt[3]{5.4} \approx 1.7544 \approx 1.75 \text{ ft.}$$

$$5H \approx 5(1.7544) \approx 8.77 \text{ ft}$$

$$w = \frac{9}{H^2} \approx 2.94 \text{ ft}$$

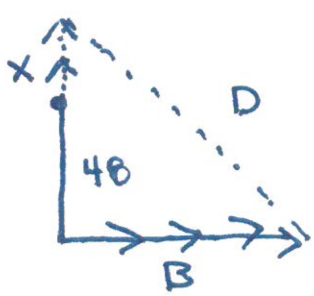
max or min?

$$S'' = 20 + \frac{216}{H^3} = +$$

$\therefore$ CONC. UP $\quad \therefore$ MIN

5.)

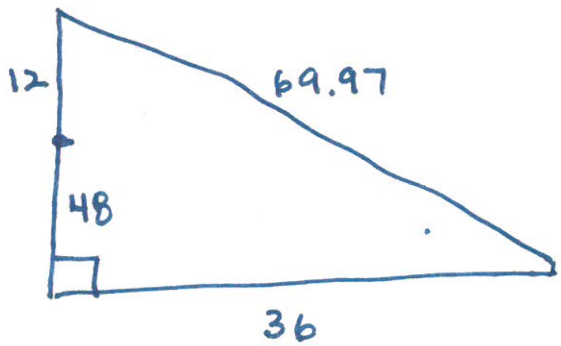
16 pts



$$\frac{dx}{dt} = 6 \frac{\text{ft}}{\text{sec}}$$

$$\frac{dB}{dt} = 18 \frac{\text{ft}}{\text{sec}} \qquad \frac{dD}{dt} = ??$$

at this instant:



$x = 12$

$B = 36$

$D = \sqrt{60^2 + 36^2} \approx 69.97$

$$(x+48)^2 + B^2 = D^2$$

DERIV:

$$2(x+48) \cdot \frac{dx}{dt} + 2B\frac{dB}{dt} = 2D\frac{dD}{dt}$$

$$\cancel{2}(12+48)\cdot(6) + \cancel{2}(36)(18) = \cancel{2}(69.97)\frac{dD}{dt}$$

$$\frac{(60)(6)+(36)(18)}{69.97} = \frac{\cancel{(69.97)}\frac{dD}{dt}}{\cancel{69.97}}$$

$$\frac{dD}{dt} = \frac{360+648}{69.97} \approx 14.41 \frac{\text{ft}}{\text{sec}}$$

7.) $f(x) = x^2 - 4x + 2$ on $[1,4]$

16pts

hypotheses: ① is $f(x)$ contin on $[1,4]$?
yes, polynomial

② is $f(x)$ diff. on $(1,4)$?
yes, deriv exists for all values in $(1,4)$

$f'(x) = 2x - 4$

$f(4) = 4^2 - 4\cdot 4 + 2 = 2$

$f(1) = 1^2 - 4\cdot 1 + 2 = -1$

(page 7)

find c, such that ....

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c - 4 = \frac{f(4) - f(1)}{4 - 1}$$

$$2c - 4 = \frac{2 - (-1)}{3}$$

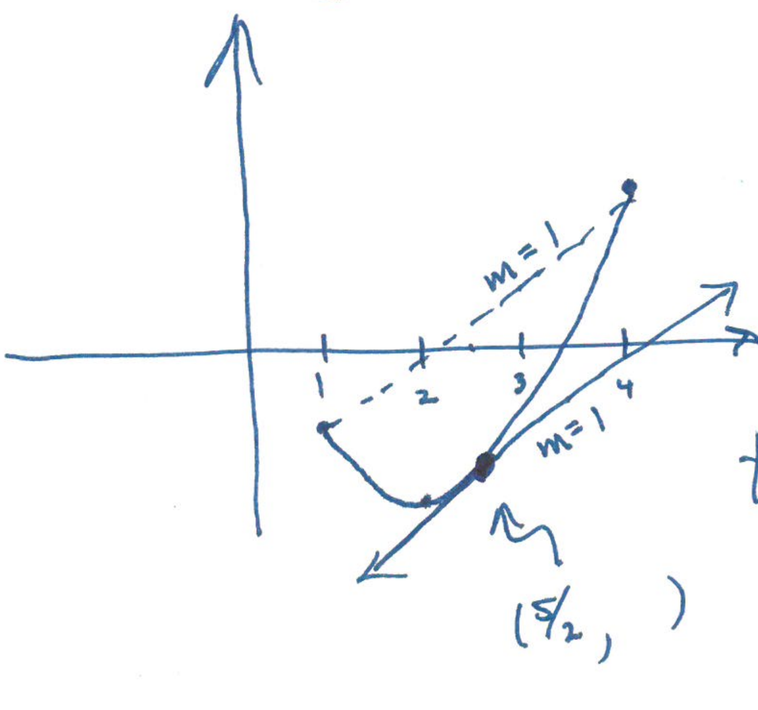
$$2c - 4 = \frac{3}{3}$$

$$2c - 4 = 1$$

$$2c - 5 = 0$$

$$c = 5/2$$

$\underline{5/2 \in (1, 4)}$



$f'(2.5) = m_{SEC}$

$f'(2.5) = 1$