NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA141-012 Test #4 Form B Wednesday, November 28, 2018 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided. Do not put work or answers on the test itself. Show all simplifications and substitutions. Do **not** use a calculator that does calculus; do not use a graphing calculator. Please include your **name, row and seat** on the front of the blue book. Put the test copy in the blue book.

1.) Integrate: $\int \frac{3\sec^2 x}{1+5\tan x}dx$

2.) Integrate and evaluate: $\int_0^6 (6x-x^2)dx$)

3.) Integrate using substitution and evaluate: $\int_0^1 5x^3(1+3x^4)^2\,dx$

4.) Integrate by parts: $\int 7x^5 \ln x\,dx$

5.) Evaluate using the Fundamental Theorem of Calculus: $\dfrac{d\ \left(\int_0^{x^4} \sqrt{1+r^3}\,dr\right)}{dx}$

6.) Using a Riemann Sum and the sigma notation formulas (provided), find the exact area under the curve $f(x)=6x-x^2$ *from* $x=0$ *to* $x=6$. (compare with #2 above)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Bonus (5 points): Integrate $\int \sec^3 t\,dt$

MA141-012 TEST #4 FORM B

TEST SOLUTIONS (16 pts each)

1.) $\displaystyle\int \frac{3\sec^2 x}{1+5\tan x}\,dx = 3\cdot\frac{1}{5}\int \frac{\sec^2 x \cdot 5}{1+5\tan x}\,dx$

let $u = 1+5\tan x$

$du = 5\cdot\sec^2 x\,dx$

$\displaystyle = \frac{3}{5}\int \frac{du}{u} = \frac{3}{5}\int \frac{1}{u}\,du = \frac{3}{5}\ln|u| + C$

$\displaystyle = \frac{3}{5}\ln|1+5\tan x| + C$

2.) $\displaystyle\int_0^6 (6x-x^2)\,dx = \left[6\cdot\frac{x^2}{2} - \frac{x^3}{3}\right]_0^6$

$\displaystyle = \left[3x^2 - \frac{x^3}{3}\right]_0^6 = \left(3\cdot 6^2 - \frac{6^3}{3}\right) - (0)$

$= 108 - 72 = 36$

3.) $\displaystyle\int_0^1 5x^3(1+3x^4)^2\,dx$

let $u = 1+3x^4$

$du = 12x^3\,dx$

$x=0 \xrightarrow{u=1+3x^4} u=1$

$x=1 \longrightarrow u=4$

$\displaystyle = 5\cdot\frac{1}{12}\int_0^1 (1+3x^4)^2\cdot \underline{x^3\cdot dx\cdot 12}$

(page 2)

$$= \frac{5}{12}\int_1^4 u^2 \cdot du = \frac{5}{12} \cdot \frac{u^3}{3}\Big]_1^4 = \frac{5}{36} u^3\Big]_1^4$$

$$= \frac{5}{36}\left[4^3 - 1^3\right] = \frac{5}{36}\left[64-1\right] = \frac{5}{36}(63) = \frac{5(7)}{4}$$

$$= \frac{35}{4} = 8\,^3/_4$$

4.) $\int 7x^5 \cdot \ln x \, dx$

let $u = \ln x$ $\qquad v = 7 \cdot \frac{x^6}{6}$

$du = \frac{1}{x} \cdot dx$ $\qquad dv = 7x^5$

$$= (u)(v) - \int v \cdot du$$

$$= (\ln x)\left(\frac{7}{6}x^6\right) - \int \left(\frac{7}{6}x^6\right)\left(\frac{1}{x} \cdot dx\right)$$

$$= \frac{7}{6}x^6 \cdot \ln x - \frac{7}{6}\int x^5 dx$$

$$= \frac{7}{6}x^6 \cdot \ln x - \frac{7}{6} \cdot \frac{x^6}{6} + C$$

$$= \frac{7}{6}x^6 \cdot \ln x - \frac{7}{36}x^6 + C$$

5.) $$\frac{d\left[\int_0^{x^4} \sqrt{1+r^3}\, dr\right]}{dx} = \sqrt{1+(x^4)^3} \cdot d(x^4)$$

$$= \sqrt{1+x^{12}} \cdot 4x^3$$

(page 3)

6.) $\text{AREA} = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot\Delta x$

$a = 0 \qquad b = 6 \qquad\qquad f(x) = 6x - x^2$

$\Delta x = \frac{b-a}{n} = \frac{6-0}{n} = \frac{6}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + i\cdot\frac{6}{n}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{6i}{n}\right)\cdot\frac{6}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(6\cdot\left(\frac{6i}{n}\right) - \left(\frac{6i}{n}\right)^2\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{36i}{n} - \frac{36i^2}{n^2}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{216i}{n^2} - \frac{216i^2}{n^3}\right)$$

$$= \lim_{n\to\infty} \left[\frac{216}{n^2} \sum_{i=1}^{n} i - \frac{216}{n^3} \sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{216}{n^2}\cdot\frac{n(n+1)}{2} - \frac{216}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[\frac{216}{2}\cdot\frac{n^2+n}{n^2} - \frac{216}{6}\,\frac{(2n^3 + \cdots)}{n^3}\right]$$

$$= \frac{216}{2}(1) - \frac{216}{6}(2) = 108 - 72 = 36$$

BONUS : 5 pts

$$\int \sec^3 t\, dt = \int \sec t \cdot \sec^2 t\, dt$$

$u = \sec t \qquad v = \tan t$

$du = \sec t \tan t\, dt \qquad dv = \sec^2 t\, dt$

$$\int \sec^3 t\, dt = (\sec t)(\tan t) - \int \sec t \cdot \tan^2 t\, dt$$

$\tan^2 t = \sec^2 t - 1$

$$\int \sec^3 t\, dt = \sec t \tan t - \int \sec t(\sec^2 t - 1)\, dt$$

$$\int \sec^3 t\, dt = \sec t \tan t - \cancel{\int \sec^3 t\, dt} + \int \sec t\, dt$$

$$+\int \sec^3 t\, dt \qquad\qquad + \cancel{\int \sec^3 t\, dt}$$

$$2\int \sec^3 t\, dt = \sec t \tan t + \int \sec t\, dt$$

$$\int \sec^3 t\, dt = \frac{1}{2}\left[\sec t \tan t + \ln|\sec t + \tan t|\right] + C$$